#1106: HyperElastic Material Models - Applications and Usage

**Product:** OptiStruct  
**Product Version:** HW 13.0 and above  
**Computer Operating System:** Windows 7, Vista_x64, Linux 64 (RHEL 5.9, RHEL 6.2, SUSE 11 SP2)  
**Categories:** Solver  
**Subcategories:** Material Models

**Topic Objective:**  
To define the material properties for non-linear hyper-elastic (Elastomeric) materials

**Topic Details**  
An elastomer is a polymer which shows non-linear elastic stress-strain behaviour. The term elastomer is often used to refer to materials which show a rubber-like behaviour.

Elastomeric materials are elastic in the classical sense. Upon unloading, the stress-strain curve is retraced and there is no permanent deformation. Elastomeric materials are initially isotropic. Figure below shows a typical stress-strain curve for an elastomeric material.

![Stress-Strain Curve](image)

Calculations of stresses in an elastomeric material requires an existence of a strain energy function which is usually defined in terms of invariants or stretch ratios

In the rectangular block in the below figure $\lambda_1$, $\lambda_2$ and $\lambda_3$ are the principal stretch ratios along the edges of the block.
In practice, the material behaviour is (approximately) incompressible, leading to the constraint equation:

\[ \lambda_1 \lambda_2 \lambda_3 = 1 \]

The strain invariants are defined as

\[ I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \]
\[ I_2 = \lambda_1^4 \lambda_2^2 + \lambda_2^4 \lambda_3^2 + \lambda_3^4 \lambda_1^2 \]
\[ I_3 = \lambda_1^4 \lambda_2^4 \lambda_3^4 \]

The hyperelastic material model:
1. is isotropic and non-linear;
2. is valid for materials that exhibit instantaneous elastic response up to large strains (such as rubber, solid propellant, or other elastomeric materials); and
3. Requires that geometric non-linearity be accounted for during the analysis step, since it is intended for finite-strain applications.

Most elastomers (solid, rubber like materials) have very little compressibility compared to their shear flexibility. In cases where the material is highly confined (such as an O-ring used as a seal), the compressibility must be modelled correctly to obtain accurate results. In applications where the material is not highly confined, the degree of compressibility is typically not crucial; for example, it would be quite satisfactory in OptiStruct to assume that the material is fully incompressible: the volume of the material cannot change except for thermal expansion.

In OptiStruct, the use of PLSOLID bulk data entry is recommended in both compressible and nearly compressible cases.

Hyperelastic materials are described in terms of a “strain energy potential,” which defines the strain energy stored in the material per unit of reference volume (volume in the initial configuration) as a function of the strain at that point in the material.

OptiStruct provides a generalized Mooney-Rivlin (Polynomial model) to model approximately incompressible isotropic elastomers.
MATHE Bulk data entry is used to define the material properties for nonlinear hyperelastic materials based on generalized Mooney-Rivlin (Polynomial) model.

**Format**

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**Field** | **Contents**
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MID | Unique material identification number. No default (Integer > 0)
Model | Specifies the type of hyperelastic material model
MOONEY - Selects the generalized Mooney-Rivlin hyperelastic model
Default = MOONEY (Character, <MOONEY, blank>)
NA | Order of the distortional strain energy polynomial function
Default = 2 (0 < Integer \leq 5)
ND | Order of the volumetric strain energy polynomial function (see comment 2).
Default = 1 (Integer)
Cpq | Material constants related to distortional deformation (Model = MOONEY).
Default = 0.0 (Real)
Dp | Material constants related to volumetric deformation (Model = MOONEY).
Default = 0.0 (Real \geq 0.0)
User can directly define the co-efficients for examples: If you choose Mooney as the HyperElastic material model C10 and C01 co-efficients need to be defined in the MATHE card.

In addition to this, the user can define the stress-strain curve using TABLES1 entry and evaluate the material co-efficients using experimental curve fit method.

The Generalized Mooney-Rivlin polynomial form of the Hyperelastic material model is written as a combination of the deviatoric and volumetric strain energy of the material. The potential \( U \) is written in polynomial form, as follows

\[
U = \sum_{p+q=1}^{N_1} C_{pq} \left( \bar{I}_1 - 3 \right)^p \left( \bar{I}_2 - 3 \right)^q + \sum_{p=1}^{N_2} \frac{1}{D_p} \left( J_{elas} - 1 \right)^{2p}
\]

Where,

- \( N_1 \) is the order of the distortional strain energy polynomial function (NA)
- \( N_2 \) is the order of the volumetric strain energy polynomial function (ND). Currently only first order volumetric strain energy functions are supported (ND=1).
- \( C_{pq} \) are the material constants related to distortional deformation (Cpq)
- \( \bar{I}_1, \bar{I}_2 \) are Invariants internally calculated by OptiStruct
- \( D_p \) are material constants related to volumetric deformation (Op). These values define the compressibility of the material.
- \( J_{elas} \) is the elastic volume strain calculated internally by OptiStruct
For a homogeneous material, homogeneous deformation modes suffice to characterize the material constants. OptiStruct accepts test data from the following deformation modes:

- Uniaxial tension and compression
- Equibiaxial tension and compression
- Planar tension and compression (also known as pure shear)

The material MATHE can be used in conjunction with PARAM, LGDISP, 1 to activate large displacement analysis with hyperelastic materials.